THERMOELASTIC BENDING RESPONSE OF ANGEL-PLY COMPOSITE PLATES RESTING ON ELASTIC FOUNDATIONS

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ABSTRACT

Different plate theories are presented to study the thermoelastic response of a multilayered angle-ply composite plate. The plate is subjected to a sinusoidal temperature and resting on different types of elastic foundations. The effects due to thermal loads and elastic foundations parameters as well as the variation of lamination angle are studied. Numerical results suggest that Pasternak's model should be used for such plates resting on elastic foundations.

Keywords: thermal load; angle-ply; composite plate; elastic foundation.

1. INTRODUCTION

Applications of advanced composite materials in structural members are increased and have stimulated interest in the accurate prediction of the response characteristics of laminated composite plates. The structural response of symmetric or anti-symmetric laminated plates has been presented over the years. Most of these articles are concerned with the cross-ply laminated plates [1-4]. Though symmetric laminated plates are simple to analyse and design, some specific applications of composite structures requires the use of anti-symmetric laminated plates to fulfill certain design requirements [5-8]. However, few publications are available in the literature for antisymmetric angle-ply composite laminated plates resting on elastic foundations.

This article deals with the thermal bending of angle-ply laminated plates resting on elastic foundations. It is to be noted that, such problems of plates resting on elastic foundations are often encountered in the analysis of the foundations of buildings, highway and railroad structures, and of geotechnical structures in general [9-12]. The effects due to several parameters such as thermal loadings, foundations, aspect and side-to-thickness ratios, as well as the lamination angle are investigated.

2. BASIC EQUATIONS

Consider a composite rectangular plate of length *a*, width *b* and thickness *h* made of an arbitrary lamination material (see Fig. 1). The plate is subjected to a temperature field *T(x,y,z)*. The plate is referred to a coordinate system *(x,y,z)* with the co-ordinates *x* and *y* along the in-plane directions and *z* along the thickness direction. The plate composed of *n* orthotropic layers oriented at angles $\theta_1, \theta_2, \ldots, \theta_n$. The material of each layer is assumed to posses one plane of elastic symmetry parallel to the *x-y* plane. Perfect bonding between the orthotropic layers and temperature-independent mechanical and thermal properties may be assumed.

The displacements of a material point in the plate are assumed as [13]

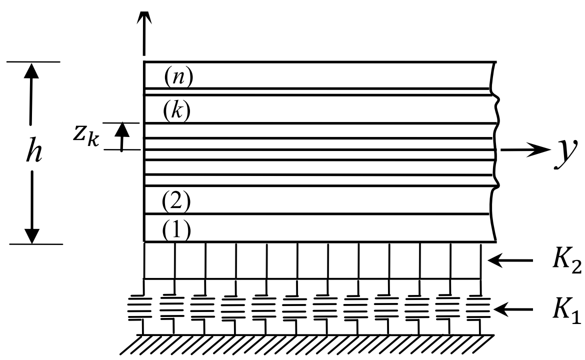

Fig. 1: Schematic diagram for the composite plate resting on elastic foundations.

$$\left.\begin{aligned} u_1 &= u(x,y) - z\frac{\partial w}{\partial x} + f(z)\psi(x,y), \\ u_2 &= v(x,y) - z\frac{\partial w}{\partial y} + f(z)\varphi(x,y), \\ u_3 &= w(x,y), \end{aligned}\right\} \quad (1)$$

where u, v, and w are the displacements of the middle surface along the axes x, y and z, respectively, and ψ and φ are the rotations about the y and x axes and account for the effect of transverse shear. The coefficient of ψ and φ is given by f and it should be odd function of z. The displacement field of classical and shear deformation theories are given by taking suitable forms for $f(z)$.

Classical plate theory (CPT): $f(z)=0$,
First-order shear deformation plate theory (FPT): $f(z)=z$,
Higher-order shear deformation plate theory (HPT): $f(z)=z\left[1-\frac{4}{3}\left(\frac{z}{h}\right)^2\right]$

Sinusoidal shear deformation plate theory (SPT): $f(z)=\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$.

In addition, the applied temperature distribution T is assumed by

$$T(x,y,z) = T_1(x,y) + \frac{z}{h}T_2(x,y) + \frac{f(z)}{h}T_3(x,y). \quad (2)$$

Also, the load-displacement relation between the plate and the supporting foundations is given by:

$$R = K_1 w - K_2 \nabla^2 w \quad (3)$$

where R is the foundation reaction per unit area, K_1 and K_2 are Winkler's and Pasternak's foundation stiffnesses, respectively, and ∇^2 represents Laplace operator. Winkler's model is simply obtained when $K_2=0$.

The six strain components ε_{ij} compatible with the displacement field in Eq. (1) are

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \quad (4)$$

The stress-strain relations for a linear elastic plate are given by

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}^k = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & c_{16} \\ c_{12} & c_{22} & 0 & 0 & c_{26} \\ 0 & 0 & c_{44} & c_{45} & 0 \\ 0 & 0 & c_{45} & c_{55} & 0 \\ c_{16} & c_{26} & 0 & 0 & c_{66} \end{bmatrix}^\kappa \begin{Bmatrix} \varepsilon_1 - \alpha_1 T \\ \varepsilon_2 - \alpha_2 T \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 - \alpha_6 T \end{Bmatrix}^k, \quad (5)$$

where $\varepsilon_1=\varepsilon_{11}$, $\varepsilon_2=\varepsilon_{22}$, $\varepsilon_4=2_{\varepsilon 23}$, $\varepsilon_5=2_{\varepsilon 13}$ and $\varepsilon_6=2_{\varepsilon 12}$. Note that, $c_{ij}^{(k)}$ are the transformed elastic coefficients and $(\alpha_1, \alpha_2, \alpha_6)$ are the thermal expansion coefficients in the plate coordinates.

The principle of virtual displacements for the present problem may be expressed as:

$$\int_\Omega \left[\int_{-h/2}^{+h/2}(\sigma_1\delta\varepsilon_1 + \sigma_2\delta\varepsilon_2 + \cdots)\mathrm{d}z + R\delta w\right]\mathrm{d}\Omega = 0, \quad (6)$$

or

$$\begin{aligned} \int_\Omega \big(&N_1\delta\varepsilon_1^0 + N_2\delta\varepsilon_2^0 + N_6\delta\varepsilon_6^0 + M_1\delta\kappa_1 + M_2\delta\kappa_2 + M_6\delta\kappa_6 + S_1\delta\eta_1 + S_2\delta\eta_2 \\ &+ S_6\delta\eta_6 + Q_4\delta\varepsilon_4^0 + Q_5\delta\varepsilon_5^0 + R\delta w\big)\mathrm{d}\Omega = 0, \end{aligned} \quad (7)$$

where

$$\begin{aligned} &\varepsilon_1^0 = \frac{\partial u}{\partial x}, \quad \varepsilon_2^0 = \frac{\partial v}{\partial y}, \quad \varepsilon_4^0 = \varphi, \quad \varepsilon_5^0 = \psi, \quad \varepsilon_6^0 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, \quad \kappa_1 = -\frac{\partial^2 w}{\partial x^2}, \\ &\kappa_2 = -\frac{\partial^2 w}{\partial y^2}, \quad \kappa_6 = -2\frac{\partial^2 w}{\partial x \partial y}, \quad \eta_1 = \frac{\partial \psi}{\partial x}, \quad \eta_2 = \frac{\partial \varphi}{\partial y}, \quad \eta_6 = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}. \end{aligned} \quad (8)$$

Note that N_l and M_ε $(i = 1,2,6)$are the basic components of stress resultants and stress couples, S_i are additional stress couples associated with the transverse shear effects and Q_j $(j=4,5)$ are transverse shear stress resultants. They can be expressed as:

$$\begin{aligned} \{N_i, M_i, S_i\} &= \int_{-h/2}^{+h/2} \{1, z, f(z)\}\,\sigma_i\,\mathrm{d}z, \\ Q_j &= \int_{-h/2}^{+h/2} f'(z)\,\sigma_j\,\mathrm{d}z, \quad (i = 1,2,6; j = 4,5). \end{aligned} \quad (9)$$

The governing equilibrium equations can be derived from Eq. (7) by integrating the displacement gradient by parts and setting the coefficients δu, δv, δw, $\delta\psi$ and $\delta\varphi$ to zero separately. Thus one obtains:

$$\left.\begin{aligned} N_{1,1} + N_{6,2} = 0, \quad N_{6,1} + N_{2,2} = 0, \\ M_{1,11} + 2M_{6,12} + M_{2,22} - R = 0, \\ S_{1,1} + S_{6,2} - Q_5 = 0, \quad S_{6,1} + S_{2,2} - Q_4 = 0. \end{aligned}\right\} \quad (10)$$

3. CLOSED-FORM SOLUTION

The determination of thermal stresses is of fundamental importance in the design of many structural components. An exact closed-form solution to Eq. (10) can be constructed when the plate is of a rectangular geometry (Fig. 1) with the following edge conditions, loading and plate constructions.

The following set of simply supported boundary conditions along the edges of the plate is considered:

$$\begin{aligned} &v = w = \varphi = N_1 = M_1 = S_1 = 0 \text{ at } x = 0,a, \\ &u = w = \psi = N_2 = M_2 = S_2 = 0 \text{ at } y = 0,b \end{aligned} \quad (11)$$

In the first place, it is to be noted that due to the macroscopic homogeneity of an anisotropic body, any translation in the x, y, or z co-ordinate direction inside the body does not alter its elastic characteristics. So, under a general co-ordinate transformation, an initially orthotropic material becomes generally anisotropic. However, there are three specific co-ordinates transformations under which an orthotropic material retains monoclinic symmetry, namely, rotations about the axes $x, y,$ or z. For example, if the material is orthotropic with respect to the old co-ordinate system, it follows under rotation through an angle θ_k about the z-axis that the transformation formulae for the stiffnesses $c_{ij}^{(k)}$ are of the form [14]:

$$\begin{Bmatrix} c_{11} \\ c_{12} \\ c_{22} \\ c_{16} \\ c_{26} \\ c_{66} \end{Bmatrix}^k = \begin{bmatrix} c^4 & 2c^2s^2 & s^4 & 4c^2s^2 \\ c^2s^2 & c^4+s^4 & c^2s^2 & -4c^2s^2 \\ s^4 & 2c^2s^2 & c^4 & 4c^2s^2 \\ -c^3s & cs(c^2-s^2) & cs^3 & 2cs(c^2-s^2) \\ -cs^3 & cs(s^2-c^2) & c^3s & 2cs(s^2-c^2) \\ c^2s^2 & -2c^2s^2 & c^2s^2 & (c^2-s^2)^2 \end{bmatrix} \begin{Bmatrix} c_{11} \\ c_{12} \\ c_{22} \\ c_{66} \end{Bmatrix}, \tag{12}$$

$$\begin{Bmatrix} c_{44} \\ c_{45} \\ c_{55} \end{Bmatrix}^k = \begin{bmatrix} c^2 & s^2 \\ cs & -cs \\ s^2 & c^2 \end{bmatrix} \begin{Bmatrix} c_{44} \\ c_{55} \end{Bmatrix},$$

where $c = \cos\theta_k$ and $s = \sin\theta_k$ and c_{ij} are the (plane stress-reduced) material stiffness of the lamina:

$$c_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad c_{12} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \quad c_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \tag{13}$$
$$c_{44} = G_{23}, \quad c_{55} = G_{13}, \quad c_{66} = G_{12},$$

in which E_i are Young's moduli in the material principal directions, ν_{ij} are Poisson's ratios, and G_{ij} are shear moduli.

To solve this problem, Navier presented the transverse sinusoidal temperature loads and deflection in the form:

$$\{T_i, w\} = \{\bar{T}_i, W\}\sin(\lambda x)\sin(\mu y), \quad i = 1,2,3, \tag{14}$$

where $\lambda = \pi/a$, $\mu = \pi/b$, $\bar{T}_i$, are constants, and W is arbitrary parameter. In addition, the remainder displacements take the forms:

$$\{u, \psi\} = \{U, \Psi\}\cos(\lambda x)\sin(\mu y),$$
$$\{v, \varphi\} = \{V, \Phi\}\sin(\lambda x)\cos(\mu y), \tag{15}$$

where U, V, Ψ and Φ are additional arbitrary parameters. The above solution form for $\{u, v, w, \psi, \varphi\}$ satisfies the simply supported boundary conditions and the parameters U, V, W, Ψ and Φ will be determined subjected to the condition that the solution satisfies the differential equations given in Eq. (10).

4. NUMERICAL EXAMPLES AND DISCUSSION

Numerical results for deflections and stresses are presented for anti-symmetric angle-ply $(\theta/3\theta)_2$ laminated plates subjected to thermal loading. The angle θ may take the values 15°, 20°, 22.5° and 25°, respectively. The plate is subjected to thermal loading as given in Eq. (2) with $T_1(\chi, y)$. All laminas are assumed to be of the same thickness and made of the same orthotropic material. The lamina properties of E-glass/epoxy are [15]:

$$E_1 = 15\,E_0, \quad E_2 = 6\,E_0, \quad G_{12} = G_{13} = 3\,E_0, \quad G_{23} = 1.5\,E_0,$$
$$\nu_{12} = 0.3, \quad \alpha_1 = 7\,\alpha_0, \quad \alpha_2 = 2.3\,\alpha_0, \quad \alpha_6 = 0.$$

where E_0 = 1 GPa and $a_0 = 10^{-6}$(1/°C). The dimensionless deflection and stresses are given by:

$$\bar{w} = \frac{wh}{\alpha_0 \bar{T}_2 a^2}, \quad \bar{\sigma}_1 = -\frac{h^2}{E_0\alpha_0\bar{T}_2 a^2}\sigma_1\left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right), \quad \bar{\sigma}_4 = \frac{h}{E_0\alpha_0\bar{T}_2 a}\sigma_4\left(\frac{a}{2}, 0, \bar{z}\right),$$

$$\bar{\sigma}_4 = \frac{h}{E_0\alpha_0\bar{T}_2 a}\sigma_4\left(\frac{a}{2}, 0, \bar{z}\right), \quad \bar{\sigma}_5 = \frac{h}{E_0\alpha_0\bar{T}_2 a}\sigma_5\left(0, \frac{b}{2}, \bar{z}\right).$$

Dimensionless deflection $\bar{w}$ and in-plane longitudinal stress $\bar{\sigma}_1$ for angle-ply $(\theta/3\theta)_2$ composite square plates are presented in Table 1. The thermal loads are assumed to be $\bar{T}_2 = \bar{T}_3 = 100$℃ and $a/h = 10$. The elastic foundation parameters $\kappa_1 = hK_1$ and $\kappa_2 = hK_2$ have different values. Three cases of elastic foundations are discussed: (1) plates without elastic foundations ($\kappa_1 = \kappa_2 = 0$), (2) plates follow Winkler's model ($\kappa_1 = 0.1$, $\kappa_2 = 0$), and (3) plates follow Pasternak's model ($\kappa_1 = 0.1$, $\kappa_2 = 0.1$). Deflection decreases as one of the foundation parameters included and as the angle of lamination θ increases. However, the in-plane longitudinal stress $\bar{\sigma}_1$ is increasing as one of the foundation parameter is included and as θ decreases. The CPT failed to get reliable results for all cases studied. The FPT gives results with higher relative errors comparing with the SPT. The HPT gives results close to those due to the SPT. The relative errors between SPT and HPT are the smallest for plates follow Pasternak's model.

Plots for deflections and transverse shear stresses of anti-symmetric, angle-ply $(\theta/3\theta)_2$ laminated plates are presented in Figures 2-4. The plate is subjected to varying sinusoidal temperature distribution with $\bar{T}_2 = 100$ ℃ and $\bar{T}_3 = 2\bar{T}_2$. Figure 2 shows the deflection $\bar{w}$ versus the side-to-thickness ratio a/h of a rectangular plate $(a/b = 3)$. Different values for the elastic foundation parameters κ_1 and κ_2 are used. The discussed cases are: (1) plates without elastic foundations ($\kappa_1 = \kappa_2 = 0$), (2) plates follow Winkler's model ($\kappa_1 = 0.04$, $\kappa_2 = 0$), and (3) plates follow Pasternak's model ($\kappa_1 = 0.04$, $\kappa_2 = 0.1$). As it is well known, CPT is independent of the side-to-thickness ratios only for plates without elastic foundations. The deflection $\bar{w}$ due to CPT decreases as a/h increases for plate follows Pasternak's or Winkler's models. For shear deformation theory, the deflection increases as a/h increases for plates without elas-

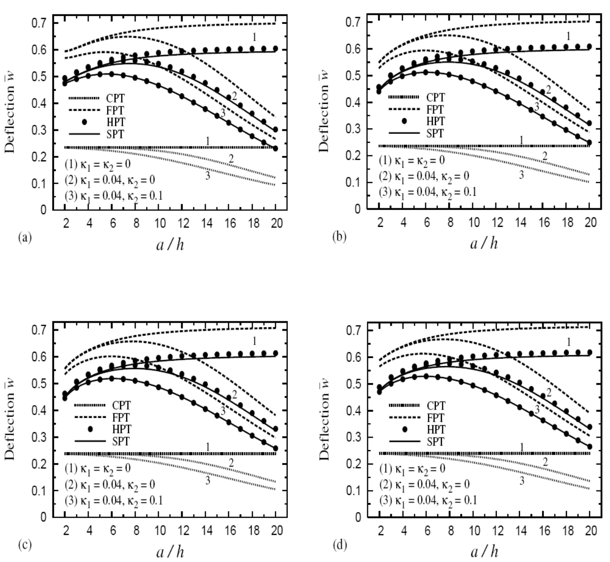

Fig. 2: The deflection $\bar{w}$ versus the side-to-thickness ratio *a/h* for angle-ply rectangular plates *(a/b = 3)* subjected to thermal loading: (a) $\theta = 15^0$, (b) $\theta = 20^0$, (c) and $\theta = 22.5^0$ (d) $\theta = 25^0$.

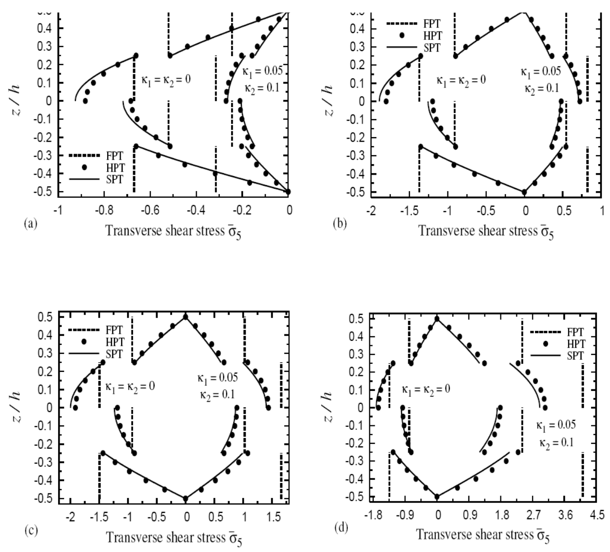

Fig. 4: Transverse shear stress $\bar{\sigma}_5$ through the thickness of angle-ply square plates subjected to thermal loading: (a) $\theta = 15^0$, (b) $\theta = 20^0$, (c) and $\theta = 22.5^0$ (d) $\theta = 25^0$.

tic foundations. However, the deflection, for plates with elastic foundations, is no longer increasing and gets smallest values for higher side-to-thickness ratios. The shear deformation theories SPT and HPT give accurate deflections compared with FPT and the errors between SPT and HPT may be omitted for plates follow Pasternak's model.

Figures 3 and 4 plot the transverse shear stresses $\bar{\sigma}_4$ and $\bar{\sigma}_5$ through the thickness of the angle-ply square plates. The thermal loads are assumed to be and the side-to-thickness ratio is *a/h = 5*. The shear stresses are very sensitive to the variation of the angle θ. In addition, $\bar{\sigma}_4$ and $\bar{\sigma}_5$ for plates follow Pasternak's model $\kappa_1 = 0.05$, $\kappa_2 = 0.1$) are greater than those for plates without elastic foundations. The FPT fails to get accurate transverse stresses through the thickness of the plate. For the transverse shear stress $\bar{\sigma}_4$ (as shown in Figure 3), the HPT compared well with SPT except at the first and third interfaces and this irrespective of the value of θ or the influence of elastic foundations. When $\theta = 15^0$, FPT may be gives reliable transverse shear stress $\bar{\sigma}_4$ within the second and third layer of the plate. When $\theta = 20^0$ or 25^0, FPT may be gives reliable transverse shear stress $\bar{\sigma}_4$ within the second and third layer of the plate that follows Pasternak's model while it gives reliable $\bar{\sigma}_4$ at the mid-plane of the plate without elastic foundations. When $\theta = 22.5^0$, FPT may give reliable transverse shear stress $\bar{\sigma}_4$ within the second and third layer of the plate that follows Pas-

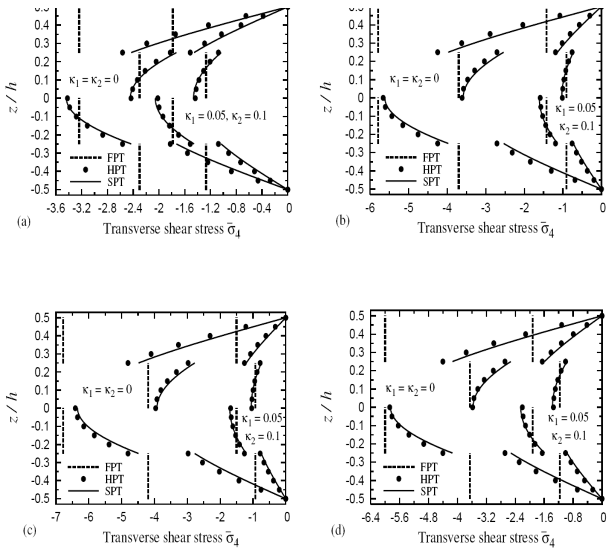

Fig. 3: Transverse shear stress $\bar{\sigma}_1$ through the thickness of angle-ply square plates subjected to thermal loading: (a) $\theta = 15^0$, (b) $\theta = 20^0$, (c) and $\theta = 22.5^0$ (d) $\theta = 25^0$.

Table 1: Dimensionless deflection $\bar{w}$ and in-plane longitudinal stress $\bar{\sigma}_4$ for angle-ply composite square plates subjected to varying sinusoidal temperature distribution

		$\bar{w}$			$\bar{\sigma}_1$		
θ	Theory	(0,0)[a]	(0.1,0)	(0.1,0.1)	(0,0)	(0.1,0)	(0.1,0.1)
15°	CPT	0.85664	0.21467	0.18701	0.18187	0.60163	0.61972
	FPT	1.70367	0.41220	0.35854	0.37570	1.18216	1.21566
	HPT	1.53334	0.37098	0.32270	0.23919	0.97065	1.00104
	SPT	1.51121	0.36564	0.31806	0.23103	0.95234	0.98230
20°	CPT	0.91010	0.22646	0.19722	0.14614	0.50849	0.52399
	FPT	1.81106	0.43528	0.37852	0.30591	1.00234	1.03107
	HPT	1.63088	0.39197	0.34086	0.19055	0.82257	0.84864
	SPT	1.60746	0.38636	0.33598	0.18369	0.80698	0.83270
22.5°	CPT	0.94306	0.23224	0.20216	0.11140	0.45198	0.46639
	FPT	1.87664	0.44593	0.38760	0.23767	0.89161	0.91828
	HPT	1.69029	0.40163	0.34910	0.13743	0.73115	0.75535
	SPT	1.66606	0.39589	0.34411	0.13171	0.71725	0.74112
25°	CPT	0.97463	0.23754	0.20669	0.07715	0.40141	0.41498
	FPT	1.93928	0.45519	0.39545	0.17008	0.79169	0.81672
	HPT	1.74684	0.41000	0.35619	0.08402	0.64862	0.67135
	SPT	1.72182	0.40415	0.35111	0.07941	0.63625	0.65867

[a] Numbers between parentheses denote the foundation parameters (κ_1, κ_2).

ternak's model only.

Figure 4 shows that FPT may give reliable transverse shear $\bar{\sigma}_5$ stress at the first and second interferences of the plate without elastic foundations when $\theta = 15^0$. The HPT may be compared well with SPT except at and near the mid-plane of the plate without elastic foundations. When $\theta = 20^0$ or 22.5^0, FPT may give accurate transverse shear stress $\bar{\sigma}_5$ at the first and third interfaces of the plate without elastic foundations. The HPT failed to get accurate values of $\bar{\sigma}_5$ near and at the mid-plane of the same plate. When $\theta = 25^0$, FPT may give reliable transverse shear stress $\bar{\sigma}_5$ at the first and third interfaces and within the second and third layer of the plate without elastic foundations. FPT failed to get any accurate transverse shear stress $\bar{\sigma}_5$ for plate follows Pasternak's model. Also, HPT gives transverse shear stress $\bar{\sigma}_5$ 1 with highly relative error compared with SPT through-the-thickness of the same plate.

5. CONCLUSIONS

Different plate theories based upon the sinusoidal shear deformation theory are used to study the thermal effects of thick multilayered angle-ply composite plates. The results of cross-ply laminated plates subjected to thermal effect and resting on elastic foundations may be considered as a special case of the present problem. Numerical results confirm that the characteristics of stresses and deflections are very sensitive to the variation of elastic foundation parameters and to the inclusion of the thermal load. The CPT is independent of nonlinear term of temperature distribution. The deflections and in-plane stresses have low sensitively to the variation of the lamination angle while the shear stresses are very sensitive to its variation.

References:

1. **Reddy, J.N. and Hsu, Y.S.,** "Effects of shear deformation and anisotropy on the thermal bending of layered composite plates", *J. Therm. Stresses,* **3**/4 (1980), 475–93.
2. **Khdeir, A.A. and Reddy, J.N.,** "Thermal stresses and deflections of cross-ply laminated plates using refined plate theories", *J. Therm. Stresses,* **14**/4 (1991), 419–438.
3. **Matsunaga, H.,** "Thermal buckling of cross-ply laminated composite and sandwich plates according to a global higher-order deformation theory", *Compos. Struct.,* **68**/4 (2005) 439–454.
4. **Zenkour, A.M.,** "Three-dimensional elasticity solutions for uniformly loaded cross-ply laminates and sandwich plates", *J. Sand. Struct. Mater.* **9**/3 (2007), 213–238.
5. **Griffin, O.H., JR.** "Three dimensional thermal stresses in angle-ply composite laminates", *J. Compos. Mater.,* **22**/1 (1988), 53–70.
6. **Swaminathan, K. and Ragounadin, D.,** "Analytical solutions using a higher-order refined theory for the static analysis of antisymmetric angle-ply composite and sandwich plates", *Compos. Struct.,* **64**/(3-4) (2004), 405–417.
7. **Swaminathan, K., Patil, S.S., Nataraja, M.S. and Mahabaleswara, K.S.,** "Bending of sandwich plates with anti-symmetric angle-ply face sheets – Analytical evaluation of higher order refined computational models", *Compos. Struct.,* **75**/(1-4) (2006), 114–120.
8. **Matsunaga, H.,** "Free vibration and stability of angle-ply laminated composite and sandwich plates under thermal loading", *Compos. Struct.,* **77**/2 (2007), 249–262.
9. **Zenkour, A.M.** "The refined sinusoidal theory for FGM plates resting on elastic foundations", *Int. J. Mech. Sci.,* **51**/(11-12) (2009), 869–880.
10. **Zenkour, A.M.,** "Hygro-thermo-mechanical effects on FGM plates resting on elastic foundations", *Compos. Struct.,* **93**/1 (2010), 234–238.
11. **Zenkour, A.M.,** "Bending of orthotropic plates resting on Pasternak's foundations by mixed shear deformation theory", *Acta. Mech. Sinica,* **27**/6 (2011), 956–962.
12. **Zenkour, A.M., Allam, M.N.M. and Sobhy, M.,** "Bending of a fiber-reinforced viscoelastic composite plate resting on elastic foundations", *Arch. Appl. Mech.,* **81**/1 (2011), 77–96.
13. **Zenkour, A.M.,** "Generalized shear deformation theory for bending analysis of functionally graded plates", *Appl. Math. Modelling,* **30**/1 (2006), 67–84.
14. **Bogdanovich, A.E. and Pastore, C.M.,** "Mechanics of Textile and Laminated Composites with Applications to Structural Analysis", London: Chapman & Hall (1996).
15. **Şahin, O.S.,** "Thermal buckling of hybrid angle-ply laminated composite plates with a hole", *Compos. Sci. Tech.,* **65**/(11-12) (2005), 1780–1790.